1) Four thieves were caught stealing so the King sent them to the dungeon. After thinking the matter over, however, the king decides to give them a chance to walk free. Thus he makes them stand in a line, one behind the other, then paints a sign on each of their backs, in either black or white. He tells the prisoners what he has done, and that there is at least one sign painted in each color. Then he tells them that they may walk free if one of them can tell him the color of the sign on their back, without turning around or attempting to peek at their own back. If any one answers incorrectly, they will all be hanged.

Assuming these are intelligent thieves, which one will answer, and how will that thief know with certainty the color of their sign?

1a) If the last thief sees all one color (white or black) in front of him he answers with the opposite color (since each color must be used once).

If the last thief doesn't answer then third thief would see in front of him and see if the backs are one color and answer with the opposite color

If the third thief doesn't answer the second thief answers with the color opposite to the one on the back of the first thief.

And so-on.

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2) So, an eccentric entrepreneur by the name of Alphonse Null has sent out a press release about his new, mind-blowing hotel: The Hotel Infinity. Null informs the world that this hotel has an infinite number of rooms (specifically, an infinity equal to the cardinality of the integers). A quick tour puts skeptics' claims to rest; as far as anyone can tell, this hotel has infinite rooms. The consequences are mind-boggling, and Null sets up a press conference to answer questions...

"So, Mr. Null, how will patrons get to their room, if their room number has, say, more digits than protons in the universe?"

"The elevators have an ingenious formula device instead of buttons... simply input the formula for your room number, with Ackermann numbers or some such... your room formula can be picked up at the front desk. There's not even any need to know what the formula means!"
"How do you produce the power and water for this hotel?"

"I have infinite generators and wells, of course. This IS an infinite hotel, you know! *chuckle*"

"What about costs? How much will it cost to stay here?"

"That's the beauty of it! Since there are as many positive even integers as there are integers, I can change the same price to only every other room and still make the same profit! I could charge only every millionth room... each guest has a one-in-a-million chance of not getting a free room, and I still get paid the same! I love the properties of infinite sets, especially when it comes to profit!"

"But, Mr. Null... I think you've made a severe mistake in your assumptions regarding profit..."

"Oh?"

The reporter then mentioned something which made Mr. Null's face turn white.

"Oh... oh goodness... THIS PRESS CONFERENCE IS OVER!" Then he ran out.

Assuming that everything Null said about the hotel is true: it really is infinite; it really is easy to get to your room; it really can generate infinite power for the guests; the cardinality of the set of multiples of a million, is the same as the cardinality of the integers...

So with what simple assumption did Mr. Null go wrong?

2a) Mr. Null knew that he had an infinite number of rooms, but assumed that he would also have an infinite number of guests. The cost of maintaining the infinite would overrun the finite income.

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3) "What's the Dominant Fifth?" asked Dr. Dingo, as his daughter Cicely came in from school.

Cicely blushed. "Just a secret society," she said. "I'm one of the vice-presidents."

"And you're meeting tonight; is that right?"

"How on earth did you know?" ask Cicely.

"You left this lying about. That's no way to keep secrets, my girl." He handed Cicely this paper:

Dominant Fifth
REASM NCNVE OTMLE SEHST TAOEI

"How did you manage to read it?" asked Cicely. "The code is known to only about eight of us."

"Change it," said Dingo. "Any fool can read that."

What did the message say, and how did Dr. Dingo decode it?

3a) ìMeet in the classroom at seven.î

Write down every 5th letter in a loop, crossing off each letter once it's been used.

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4) John and Adam just landed a Saturday detention with Dr. Taxicab, the crazy math teacher (don't ask what they were doing). When John and Adam got to the detention room at 7:00 AM, they saw, on the giant whiteboard, the numbers 1 through 1729 written out, and a note with the following instructions:

†

"Each of you will take turns erasing two numbers and replacing them with the positive difference of the two erased numbers. You will continue until only one number remains on the board. John will win if the number remaining is odd, and Adam will win if the number remaining is even."

†

Assume that John and Adam do not play any strategy (they just want to get out as fast as possible, so they pick random numbers). What is the probability that Adam wins the game?

4a) Adam can never win. Total sum parity never changes and is initially odd.

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5) Prove that 7 + 7 = 12.

5a) 12 = XII
7 = VII

Cut XII in half horizontally and it reveals two sets of VII

---

6) A post sign on the highway indicates the following distances to four destinations. From the information given in the first three, can you determine the missing info?

Memphis ---> 2001
Delos ---> 550
Halifax ---> 49
Medeira ---> ?

6a) 1501 calculated by the Roman numerals in each word (MDI)

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7) Given the following information, what is 10 + 10?

1 + 1 = 0
2 + 2 = 0
3 + 3 = 0
4 + 4 = 2
5 + 5 = 0
6 + 6 = 2
7 + 7 = 0
8 + 8 = 4
9 + 9 = 2
10 + 10 = ?

7a) 2, count the number of holes in each number

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8) You're a cyborg in a pistol duel with two other cyborgs:

* You have been programmed to fire pistols with an accuracy of 33%.
* The other two cyborgs shoot with accuracies of 100% and 50% respectively.
* The rules of the duel are one shot per cyborg per round.
    * The shooting order is from worst shooter to best shooter.
        * Thus, you go first, the 50% shooter goes second, and the 100% shooter fires third.
    * Then the round repeats.
    * If a cyborg dies, its turn is skipped.

What should you shoot at in round one to maximize your chances of survival over time?

8a)

* A ñ 1/3 accuracy (33%) (You)
* B ñ 1/2 accuracy (50%)
* C ñ 1 accuracy (100%)
* X * Y = X shoots at Y

Options:

1. A * B.
2. A * C.
3. A * e (A shoots at nothing ñ deliberately misses)

Process:

1. A * B
    A hits B (1/3 of the time).
        C * A and kills A.
    A misses B (2/3 of the time).
        B * C (C is B's largest threat).
            B kills C (1/2 of the time).
                A * B for resolution.
            B misses C (1/2 of the time).
                C * B (100% kill)
                    A * C
                        A kills C (1/3)
                        A misses C (2/3)

A * B (dead C):

= (1/3) + [(2/3) * (1/2) * (1/3)] + [(2/3) * (1/2) * (2/3) * (1/2) * (1/3)] + ...

= (1/3) + (1/3)n * (1/3)

= (1/3) + (1/6) = (1/2)

A * B (with C):

= [(2/3) * (1/2) * (1/2)] + [(2/3) * (1/2) * (1/3)]

= 10/36 chance of winning.

2. A * C

A hits C (1/3)

B * A ñ simplifies down to A * B (dead C) case.

A misses C (2/3)

B * C

B kills C (1/2)

Simplifies to A * B (dead C) case.

B misses C (1/2)

C kills B.

(1/3) chance to kill C. Else A is killed.

A * C

= [(1/3) * (1/2) * (1/2)] + [(2/3) * (1/2) * (1/2)] + [(2/3) * (1/2) * (1/3)]

= (1/12) + (1/6) + (1/9)

= 13/36 chance of winning.

3. A * e

B * C

B kills C (1/2)

Simplifies to A * B (dead C)

B misses C (1/2)

C kills B.

(1/3) chance to kill C. Else A is killed.

A * e

= [(1/2) * (1/2)] + [(1/2) * (1/3)]

= (1/4) + (1/6)

= 15/36 chance of winning.

Best option at 15/36 (41.7%) chance of winning is to miss the first shot purposely.

Comedy option: you miss your deliberate miss and accidentally kill both cyborgs

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9) You are at a track day at your local racecourse in your new Porsche. Because it's a crowded day at the track, you are only allowed to do two laps. You haven't driven your car at the track yet, so you took the first lap easy, at 30 miles per hour. But you do want to see what your ridiculous sports car can do. How fast do you have to go on the second lap to end the day with an average speed of 60 miles per hour?

9a) Infinity (impossible).
Let C = length of track, and X be the speed of the second lap.

Lap 1:

(C/30 mph)

Lap 2:

(C/X mph)

Lap Total:

{2C/[(C/30) + (C/X)]} = 60 mph
{[(C/30) + (C/X)]/2C} = (1/60)
(1/60) + (1/2X)= (1/60)
(1/2X) = 0
(1/2) * (1/X) = 0
(1/X) = 0
X = (inf) mph